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LETTER TO THE EDITOR

Fractal properties of the percolating backbone in three dimensions

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Abstract. A recursive algorithm to determine the backbone of the two-dimensional percolating cluster is adjusted to measure the fractal properties of the backbone of the three-dimensional percolating cluster. The specific properties measured are the fractal dimensionality of the backbone, $d_f^{BB} = 1.75 \pm 0.04$, the chemical dimensionality, $d_l = 1.26 \pm 0.03$, and the minimum path dimensionality $d_{min} = 1.39 \pm 0.03$.

Recently developed algorithms have been used to measure the fractal properties of the percolating backbone in two dimensions (Laidlaw *et al* 1987, Puech and Rammal 1983) and in both two and three dimensions (Herrmann *et al* 1984). The two-dimensional results reported by Laidlaw *et al* are in excellent agreement with other existing numerical values (see, e.g., table 1 of Hong and Stanley (1983) for a summary of numerical results prior to 1983). Sahimi (1984) postulated a scaling relation for the backbone exponent, β_B

$$B(p) - (p - p_c)^{\beta_B} \qquad p > p_c \tag{1}$$

where p is the fraction of occupied sites, p_c the percolation threshold and $B(p)$ the number of sites belonging to the backbone. The scaling relation is

$$\beta_B = \tfrac{1}{2}(\nu d + 3\beta) - 1 \tag{2}$$

where ν is the correlation length exponent, β describes how the fraction of sites belonging to the infinite cluster varies where $(p - p_c)$, and d is the dimensionality of Euclidean space. The fractal dimensionality of the backbone, d_f^{BB}, is related to β_B by $d_f^{BB} = d - \beta_B/\nu$ (Kirkpatrick 1978, Stauffer 1985) and for two dimensions the predicted value is 1.594, whilst the value reported by Laidlaw *et al* is 1.61 ± 0.01. The corresponding prediction for three dimensions is ~ 1.91.

The random-site percolation cluster is grown by the Leath algorithm (Leath 1976) on the simple cubic lattice at $p_c = 0.3117$ (Heermann and Stauffer 1981). The recursive algorithm to determine the backbone is implemented in two steps. In the first the cluster is described in terms of a tree-like structure and a path constructed between the endpoints. The second step constructs the backbone by adding linear segments of the cluster to the growing backbone. The segments added are those that can be attached to the growing backbone at two distinct points. This procedure, which is implemented in a recursive manner, is continued until no further growth is possible. Further details of the algorithm may be found in Laidlaw *et al* (1987).

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Figure 1 shows the variation of the mass (M) and radius of gyration (R_g) with chemical distance t (Middlemiss *et al* 1980) for values of t from 10 to 80.

The following exponents are defined:

$$M \sim t^{d_t} \qquad R_g \sim t^{\tilde{\nu}} \qquad (\tilde{\nu} = 1/d_{\min})$$

$$M \sim R_g^{d_f^{BB}} \qquad d_f^{BB} = d_t/\tilde{\nu}.$$

A least-squares fit of the data yielded $d_t = 1.26 \pm 0.03$, $\tilde{\nu} = 0.72 \pm 0.01$ and $d_f^{BB} = 1.75 \pm 0.04$. The error bars were determined by observing the effects of including the smallest value of t. The variation of M and R_g with t shows no systematic finite-size effects for $t > 15$ and the error bars are perhaps reliable. However, it is impossible to exclude, as with all simulations, that larger systems may lead to a new systematic trend.

The fractal dimensionality of the backbone is in excellent agreement with the direct measurement of Herrmann *et al* (1984) ($d_f^{BB} = 1.77$) and also with the extrapolated result of Herrmann and Stanley (1984) from the measurement of volatile fractals ($d_f^{BB} = 1.74$). In three dimensions we do not find the same measure of agreement with Sahimi ($d_f^{BB} = 1.91$). The value obtained for $d_{\min}$ is in good agreement with the results of Herrmann *et al* (1984) ($d_{\min} = 1.35$).

To summarise, we have applied the algorithm of Laidlaw *et al* to construct the backbone of the percolation cluster in three dimensions. We find $d_f^{BB} = 1.75 \pm 0.04$, $d_t = 1.26 \pm 0.03$ and $d_{\min} = 1.39 \pm 0.03$.

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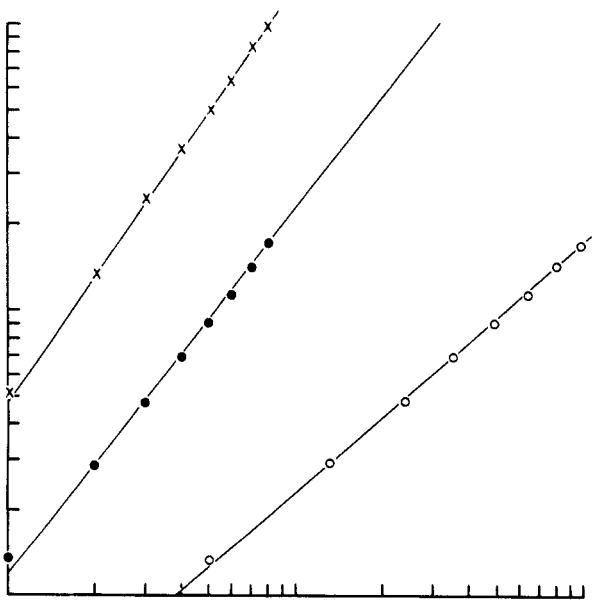

Figure 1. Log-log plot of the variation of mass (●) and radius of gyration squared (×) with chemical distance (10-80). The respective slopes lead to a chemical dimensionality, d_t, of 1.26 ± 0.03 and a minimum path dimensionality, $d_{\min}$, of 1.39 ± 0.03. The variation of mass with radius of gyration squared (1-100) is shown by (○) and the corresponding fractal dimensionality, d_f^{BB}, is 1.75 ± 0.04.

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